Introduction

In this write up, we will talk about the relationship between absolute pressure and altitude. We will talk about various ways to estimate the altitude given an absolute pressure. We will show you the best way (our own proprietary formula) to accurately estimate altitude given absolute pressure. Finally, we will provide you with the comprehensive table supplied by NASA showing various pressure at altitude up to 250,000 feet.

There is a PDF copy as well as the Altitude vs Pressure and Vacuum Table that you are welcome to download and use and we would be grateful if you'd give us credit. I hope you'll enjoy reading this write up as much as we have enjoyed writing it and building altitude simulation systems for you. As always, feedback, questions, and comments are welcome.

Altitude vs Pressure and Vacuum

Earth's Atmosphere is composed of various Gasses; mostly Oxygen and Nitrogen. Just as any object, these gas molecules are being constantly pulled by the earth's gravity towards the ground. Due to this pull, the molecules at sea level have a higher pressure applied on by all the molecules above it who are also being pulled by gravity towards the ground. This also means that the molecules that are 100 meters above sea level have pressure applied on them by the air molecules above them, but not below them. The higher up you go, the less pressure is applied by air molecules as the air gets thinner. This is the reason why the air pressure is higher at sea level as compared to 20,000 ft above sea level. By taking a pressure reading at various altitudes, one can plot the relationship between absolute pressure and altitude and derive a formula. Once this relationship is established, one can now convert it backwards from absolute pressure to altitude as these now become the same things expressed either in meters or feet or absolute pressure. Keep in mind that this relationship only applies to planet earth. Furthermore, the atmospheric pressure is not uniform across each elevation, there are High- and Low-pressure pockets across the atmosphere due to weather. However, the relationship between absolute pressure and altitude is a good approximation and valid for testing and simulation purposes.

Illustrating relationship between Altitude and Absolute Pressure

Take a look at the image which is divided into three columns. The first column is the individual force, this is the force of gravity applied to each air molecule by earth. The second column is the altitude in meters, this can also be illustrated in feet. The third column is the cumulative force applied to all air molecules at a specific altitude.

Keep in mind that this is simply a model. There is no such thing as a perfect model and that models are used to summarize the laws nature.

With that said, let's take the air molecule at 15km altitude, notice that there is only one ARROW of force applied to it and no ARROWS of force applied above it? This means that the unit of Pressure is one. Going down to 10km of altitude, notices that individual force applied is still one ARROW, however, the Cumulative pressure is now two arrows because one arrow is from the earth's gravity and the second arrow is from the air molecules pushing down on it from 15km altitude.

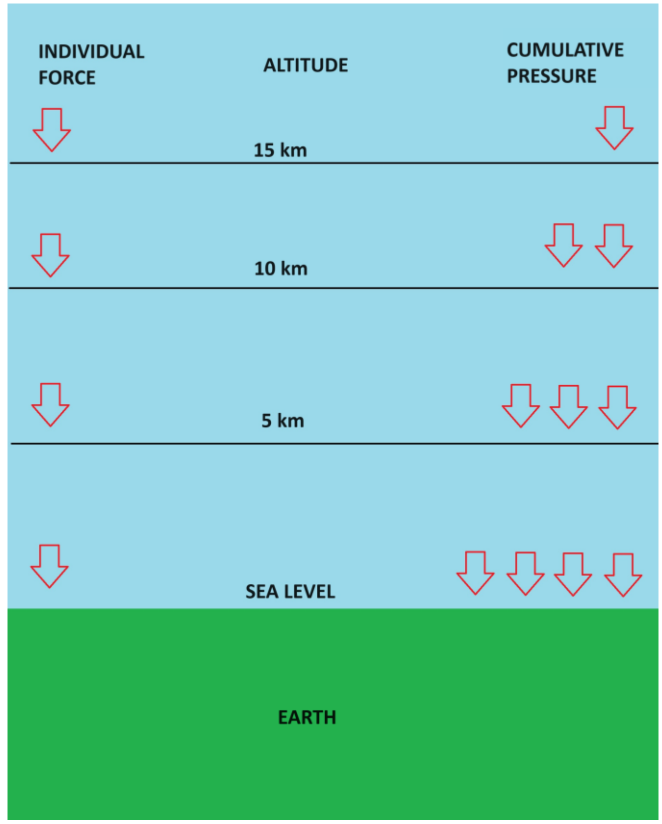

Figure 1. Illustration of Atmospheric Pressure

As we move down to sea level, we still have one ARROW of pressure applied to the molecule at sea level, however, the cumulative pressure is FOUR Arrows which is the sum of all individual molecules pushing down on it. Pressure is Force over a surface area and is denoted in units of pounds per square inch (psi) or Pascal which is Newton per Square Meter (Pa). Pressure is also sometimes denoted in length of water or mercury. Such as inches of water or inches of Mercury (inHg) or millimeters of Mercury (Torr). The reason being that pressure can be expressed via depth of water or mercury is the same reason that air pressure is a function of altitude. The medium doesn't matter, as long as gravity is acting on the molecules, the molecules at the bottom will experience a higher cumulative force due to the force applied by all molecules above it.

The US Standard Atmosphere 1976 Document

The so-called "US Standard Atmosphere 1976" is a 250-page document created by NASA, the National Oceanic and Atmospheric Administration, and the US Airforce. Further contributors include countless scientists, professors, and engineers from a wide range of Universities, Research institutions, Private Corporations, and various other Organizations. This is truly a magnificent document! (I know I sound like a total nerd)

The number of resources, brainpower, and research-hours that have gone into this document is astonishing and nothing short of a gift to us because we get to just access it from the palm of our hands. And I know, I am not that naïve, I know that this document was created during the "space race" in order to provide atmospheric data to rocket companies so that we can bomb each other from an increasingly farther distance. This is quoted in the document itself, quote: "... To provide the then newborn missile industry with a realistic description of the atmosphere ..." you catch the drift.

Fortunately, the data contained in this document is not only beneficial to rocket scientists, it is also beneficial to the medical industry, to the food industry, the aerospace industry, research, etc.… As you can see, not everything should be looked at in binary, black and white terms. And neither should our earth's atmosphere. Because our atmosphere is not static and it does not exactly equal 760 Torr at sea level. You have probably noticed that we experience High-and-Low-Pressure fronts at sea level causing the pressure at sea level to vary between 780 Torr to as low as 720 Torr. That is a wide range which is why the US Standard Atmosphere document was created to standardize and create a generally accepted altitude vs pressure graph (among other things).

If you ever start reading this document, you will learn that measuring and quantifying the earth atmosphere is not as easy as it sounds. And as I mentioned before, the sketch with arrows illustrating the concept of why the pressure is higher at sea level is a highly simplified version of this. The earth atmosphere is affected by temperature, pressure, weather conditions, seasons, even solar cycles! The air pressure at a specific altitude will never be the same due to various conditions and it will vary widely. All data derived are only best estimates, and as quoted by the document: "… A hypothetical vertical distribution of atmospheric temperature, pressure and density which, by international agreement, is roughly representative of year-round, midlatitude conditions..."

Reading through this document made me realize how fascinating our atmosphere is while at the same time how it can be used to properly package medical equipment as well as chocolate, candies, as well as cheeseburger patties. To conclude, the most fascinating thing I found is googling all the contributors' names listed in this document and discovering the lives they lived. I was impressed that many of them have their own Wikipedia page, some died in the 1970's while others just recently passed away in the 2020's. Some even have mountains, rivers, college buildings, and rockets components named after them.

Sanatron
www.sanatron.com
426 South 1000 East STE708
Salt Lake City, UT 84102
Web: www.sanatron.com
Email: info@sanatron.com
Phone: (801) 876-0831

Geometric vs Geopotential Altitude

NASA's "US Standard Atmosphere 1976" references two different altitudes in its table which are the Geometric and Geopotential Altitude.

Geometric Altitude refers to the straight-line distance from sea level to the object of interest.

Geopotential Altitude, on the other hand, is a mathematically adjusted altitude to account for the change in gravity as gravity is different at various altitudes. Geopotential Altitide is a potential energy adjusted altitude. Furthermore, even at sea level, gravity does not remain the same across the world. It varies slightly due to uneven mass and density at that location. This in turn means that the absolute air pressure will vary.

In the comprehensive table, we will use Geopotential altitude. We will do this to standardize the altitude vs. pressure across the globe. Furthermore, when converting altitude to absolute pressure, the difference between the geometric and geopotential altitude is negligible at +/- 0.05% this is much less than the mathematical formula used to calculate altitude.

Moreover, the pressure is also a function of temperature and temperature across altitudes is not the same for earth's atmosphere; which is another reason why we are using geopotential altitude over geometric altitude.

Altitude vs Absolute Pressure Formula and Estimations

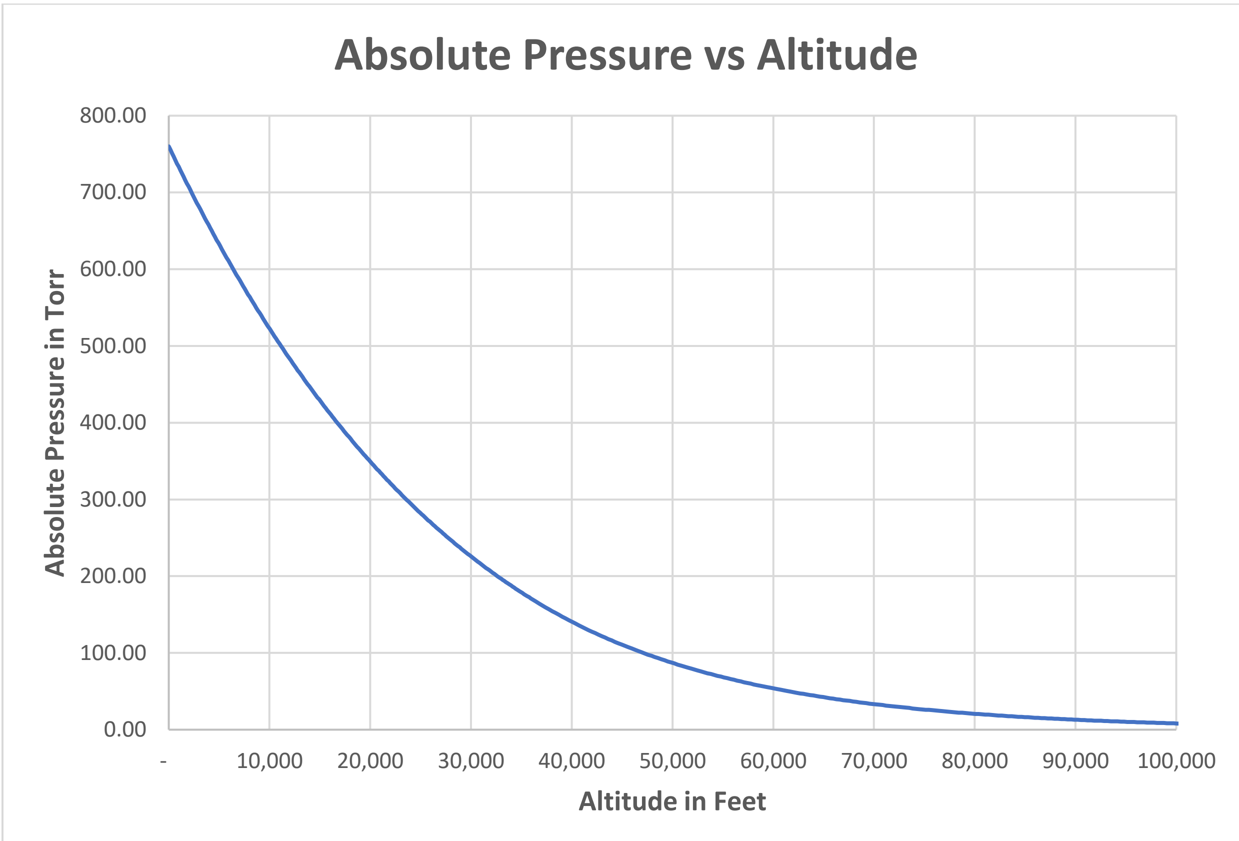

Figure 2. Absolute Pressure vs. Altitude Graph

There are some challenges when trying to find a good formula to accurately calculate the absolute pressure at specific altitude. The reason for this is because our earth's atmosphere is not uniform (gravity, air temperature, density, humidity, etc.) and therefore finding a uniform formula will be a challenge. Keep in mind that the earth's atmosphere is segmented into Troposphere (below 11km), Stratosphere (11km to 51 km), Mesosphere (51km to 71km), and Ionosphere (71km and above).

In addition to the non-uniformity of earths atmosphere, there are also variations in absolute pressure at sea level due to weather conditions. While it is generally accepted that the absolute pressure at sea level to be equal to 760 Torr, at low pressure conditions, the absolute pressure at sea level can drop to as low as 720 Torr. On the other hand, at high pressure or fair-weather conditions, the absolute pressure can rise to as high as 780 Torr. This corresponds to a variance in absolute pressure of 8% at sea level simply due to weather conditions.

This variance exists across the various altitudes of earth's atmosphere. Let's say you are looking at an altitude of 20km where the absolute pressure is 41Torr. If we apply the 8% variance you would have 39.36 Torr and 42.64 Torr which corresponds to an altitude of 20.3km and 19.8km respectively. Quite a large altitude difference of 500 meters.

Therefore, the relationship between absolute pressure and altitude is only a best estimate model and should never be taken too accurately.

To estimate the pressure at specific altitude, a good estimate is to use Pascals Law:

$$P(h) = P_o * e^{\left(\frac{-g_o * M * h}{R * T}\right)}$$

Where:
P(h) = Calculated Pressure at height (h)
P_o = Pressure at Sea Level (101.3kPa or 760 Torr)
$-g_o$ = Gravitational constant (9.81 m/s^2)
M = molar mass of air (0.02896 kg/mol)
h = height (in meters)
R = Universal Gas constant (8.314 N*m/(mol*K))
T = Temperature in kelvin (288 Kelvin)

Solving for height (h) and plugging in the numbers you get the following formula:

$$h\ (meters) = \frac{\left[\ln\left(\frac{P}{P_o}\right)\right]}{-0.00011865}$$

There are other formulas which take into account the drop of air density and temperature at height where the Pressure at altitude is calculate by:

$$P\ (h) = \ P_o * (1 - 0.000025577 * h)^{5.25588}$$

Solving for height (h) and plugging in the numbers you get the following formula:

$$h(meters) = \frac{\left(1 - [\left(\frac{P}{P_o}\right)^{\left(\frac{1}{5.25588}\right)}]\right)}{0.0000225577}$$

Unfortunately, the equations above will give too high of an error. The average error their formula will give you is +16% meaning it will calculate the altitude to be on average 16% higher than actual. The error range is +0% to +25.4%. In some instances, it will give you a whopping 25% higher calculated Altitude than actual.

Sanatron Altitude vs Pressure Calculation and Conversion

Finally, we introduce our own proprietary formula which we utilize to convert absolute pressure to altitude in feet. We do this by second-order curve fitting the NASA "US Standard Atmosphere 1976" table values and by dividing by the curve fit error which is also curve fitted. The result is an astonishing average 0% average error with the discretized error band at -3.7% and +1.4%. This is an order of magnitude better than what the other websites give you and we are happy that you have found this write up because it will save you from large altitude calculation errors!

The altitude vs Pressure is given by the following formula:

$$Altitude(meters) = \frac{-\left(\frac{\log\left(\frac{P}{P_o}\right)}{0.0000515}\right)}{(0.000000394 * P^2 - 0.000603678284 * P + 1.24169838380512)}$$

Here is the Microsoft Excel Version that you are welcome to copy and paste
Altitude (in meters):

= (-1*((LOG(P/Po)/0.0000515)))/(0.000000394*P*P - 0.000603678284*P + 1.24169838380512)

Where:
P = Measured Pressure
P_o = Pressure at Sea Level (760 Torr)
Note: The formula above is only valid for units in Torr.

Sanatron
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426 South 1000 East STE708
Salt Lake City, UT 84102
Web: www.sanatron.com
Email: info@sanatron.com
Phone: (801) 876-0831

Comparing the performance of the formulas

Looking at the table, we will compare the three Altitude Estimating Formulas such as the Pascals Law, Engineering Toolbox, and Sanatron Formula. All three formulas estimate the altitude fairly decently at below 10km. However, after we increase the altitude above 10 km, the errors in Pascals Formula start increasing while the Engineering Toolbox formula completely breaks down. Only the Sanatron Proprietary Formula gives a good altitude estimate.

Table 1. Comparing Formula Performance

US Standard Atmosphere Pressure (Torr)	US Standard Atmosphere Altitude (meters)	US Standard Altitude (feet)		Pascals Law Formula (meters)	Absolute Error (%)	Engineering Toolbox Formula (meters)	Absolute Error (%)	Sanatron Proprietary Formula (meters)	Absolute Error (%)
674.11	1,000	3,281		1,011	1.1%	1,000	0.0%	998	0.2%
596.26	2,000	6,562		2,045	2.3%	2,000	0.0%	2,002	0.1%
462.33	4,000	13,124		4,189	4.7%	4,000	0.0%	4,004	0.1%
268.02	8,000	26,248		8,784	9.8%	7,974	0.3%	7,931	0.9%
77.162	16,000	52,496		1,9279	20.5%	15,643	2.2%	16,109	0.7%
21.98	24,000	78,744		29,862	24.4%	21,740	9.4%	24,319	1.3%
6.51	32,000	104,992		40,118	25.4%	26,409	17.5%	32,429	1.3%
0.56	50,000	164,050		60,793	21.6%	33,093	33.8%	49,001	2.0%
0.013	76,000	249,356		92,508	21.7%	38,839	48.9%	74,544	1.9%

Reference:

https://www.sanatron.com/articles/altitude-vs-pressure-and-vacuum.php

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